Review: Complex Numbers

The extended system of numbers, called the *complex numbers*, consists of all real numbers and sums of real numbers and real number multiples of i. The following are all examples of complex numbers:

$$-6, \quad 5i, \quad \sqrt{5}, \quad -7i, \quad \frac{5}{2}i + \frac{2}{3}, \quad -2 + 3i, \quad 5 - 3i, \quad \frac{1}{3} + \frac{4}{5}i$$

DEFINITION Complex Number

A **complex number** is any number that can be written in the form

$$a + bi,$$

where a and b are real numbers. The real number a is the **real part**, the real number b is the **imaginary part**, and $a + bi$ is the **standard form**.

SECTION P.6 Exercises

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1–8, write the sum or difference in the standard form $a + bi$ without using a calculator.

1. $(2 - 3i) + (6 + 5i)$
2. $(2 - 3i) + (3 - 4i)$
3. $(7 - 3i) + (6 - i)$
4. $(2 + i) - (9i - 3)$
5. $(2 - i) + \left(3 - \sqrt{-3}\right)$
6. $\left(\sqrt{5} - 3i\right) + \left(-2 + \sqrt{-9}\right)$
7. $(i^2 + 3) - (7 + i^3)$
8. $\left(\sqrt{7} + i^2\right) - \left(6 - \sqrt{-81}\right)$

In Exercises 9–16, write the product in standard form without using a calculator.

9. $(2 + 3i)(2 - i)$
10. $(2 - i)(1 + 3i)$
11. $(1 - 4i)(3 - 2i)$
12. $(5i - 3)(2i + 1)$
13. $(7i - 3)(2 + 6i)$
14. $\left(\sqrt{-4} + i\right)(6 - 5i)$
15. $(-3 - 4i)(1 + 2i)$
16. $\left(\sqrt{-2} + 2i\right)(6 + 5i)$

In Exercises 17–20, write the expression in the form bi, where b is a real number.

17. $\sqrt{-16}$
18. $\sqrt{-25}$
19. $\sqrt{-3}$
20. $\sqrt{-5}$

In Exercises 21–24, find the real numbers x and y that make the equation true.

21. $2 + 3i = x + yi$
22. $3 + yi = x - 7i$
23. $(5 - 2i) - 7 = x - (3 + yi)$
24. $(x + 6i) = (3 - i) + (4 - 2yi)$

In Exercises 25–28, write the complex number in standard form.

25. $(3 + 2i)^2$

26. $(1 - i)^3$

27. $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^4$

28. $\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)^3$

In Exercises 29–32, find the product of the complex number and its conjugate.

29. $2 - 3i$

30. $5 - 6i$

31. $-3 + 4i$

32. $-1 - \sqrt{2}i$

In Exercises 33–40, write the expression in standard form without using a calculator.

33. $\frac{1}{2 + i}$

34. $\frac{i}{2 - i}$

35. $\frac{2 + i}{2 - i}$

36. $\frac{2 + i}{3i}$

37. $\frac{(2 + i)^2(-i)}{1 + i}$

38. $\frac{(2 - i)(1 + 2i)}{5 + 2i}$